2018-2019 Curriculum Guide

April 19, 2019- End of School Year

Math in Focus

Unit 4: Foundations of Multiplication Graphs & Shapes

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit 4 Overview
Chapter 17 & 19
Eureka Module 6
Eureka Module 7 Topic A

- Students will learn how to analyze more complex picture graphs, bar graphs, and line plots that involve symbols that may represent more than one item.
- Studnets will learn to solve word problems using the data they find in the picture graphs,bar graphs, and line plots.
- Learn about new plane shapes , such as trapezoid, hexagon, quadrilateral, and pentagon.
- Draw and copy plane shapes and figures and extend these drawings to build models by combining solid shapes.
- Students learn the following interpretations of even numbers:

  1. A number that occurs when skip-counting by twos is even: 2, 4, 6, 8, …

  2. When objects are paired up with none left unpaired, the number is even.

  3. A number that is twice a whole number (doubles) is even.

  4. A number whose last digit is 0, 2, 4, 6, or 8 is even.

- Armed with an understanding of the term even, students learn that any whole number that is not even is called odd and that when 1 is added to or subtracted from an even number, the resulting number is odd.
- Students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit. Hence, they begin their experience toward understanding that any unit may be counted (e.g., 3 dogs, 3 tens, or even 3 fives).

Unit 4: Foundations of Multiplication, Graphs, & Shapes		
Module 6: Foundations of Multiplication and Division		
Topic	**Lesson**	**Student Lesson Objective/ Supportive Videos**
Topic A: Formation of Equal Groups	Lesson 1&2	Use manipulatives to create equal groups. Use math drawings to represent equal groups, and relate to repeated addition https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 4	Represent equal groups with tape diagrams, and relate to repeated addition https://www.youtube.com/watch?v
Topic B: Arrays and Equal Groups	Lesson 5	Compose arrays from rows and columns and count to find the total using objects. https://www.youtube.com/watch?v
	Lesson 6	Decompose arrays into rows and columns, and relate to repeated addition https://www.youtube.com/watch?v
	Lesson 7	Represent arrays and distinguish rows and columns using math drawings https://www.youtube.com/watch?v
	Lesson 9	Solve word problems involving addition of equal groups in rows and columns https://www.youtube.com/watch?v
Mid-Module Assessment Task		
Topic C: Rectangular Arrays as a Foundation	Lesson 10&11	Use square tiles to compose a rectangle and relate to the array model. https://www.youtube.com/watch?v https://www.youtube.com/watch?
	Lesson	Use math drawings to compose a rectangle with square tiles

For Multiplication and Division	12	https://www.youtube.com/watch?v
	Lesson 13	Use square tiles to decompose a rectangle https://www.youtube.com/watch?v
	Lesson 14	Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares https://www.youtube.com/watch?v
	Lesson 15	Use math drawings to partition a rectangle with square tiles, and relate to repeated addition https://www.youtube.com/watch?v
	Lesson 16	Use grid paper to create designs to develop spatial structuring https://www.youtube.com/watch?v
Topic D: The Meaning of Even and Odd Numbers	Lesson 17	Relate doubles to even numbers and write number sentences to express the sums. https://www.youtube.com/watch?v
	Lesson 18	Pair objects and skip-count to relate to even numbers https://www.youtube.com/watch?v
	Lesson 19	Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers https://www.youtube.com/watch?v
	Lesson 20	Use rectangular arrays to investigate odd and even numbers https://www.youtube.com/watch?v
End-Module Assessment Task		

Topic	Activity	Standard
MIF **Chapter 17** Picture Graphs	Chapter Opener	2.MD.10
	Lesson 1: Reading Picture Graphs and Scales	2.MD.10
	Lesson 2: Making Picture Graphs	2.MD.10
	Lesson 3: Real- World Problems: Picture Graphs	2.MD.10
	Lesson 4: Bar Graphs and Line Plots	2.MD.9-10

Eureka Module 7 Topic A : Problem Solving with Categorical Data		
Topic	**Lesson**	**Student Lesson Objective/ Supportive Videos**
Topic A: Problem Solving with Categorical Data	Lesson 1	Sort and record data into a table using up to four categories; use category counts to solve word problems. https://www.youtube.com/watch?v
	Lesson 2	Draw and label a picture graph to represent data with up to four categories. https://www.youtube.com/watch?v
	Lesson 3 &4	Draw and label a bar graph to represent data; relate the count scale to the number line. Draw a bar graph to represent a given data set https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 5	Solve word problems using data presented in a bar graph https://www.youtube.com/watch?v

Topic	**Activity**	**Standard**
MIF **Chapter 19** Shapes and Patterns	Chapter Opener	2.G.1
	Lesson 1: Plane Shapes (3 Days)	2.G.1
	Lesson 2: Solid Shapes	2.G.1
	Lesson 3: Making Patterns (2 Days)	2.G.1

New Jersey Student Learning Standards: Numbers and Operations in Base Ten	
2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Second graders apply their work with doubles to the concept of odd and even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends or doubles addition facts (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example: **Is 8 an even number? Justify your thinking.**

Student A

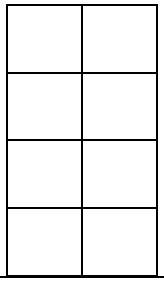
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.

Student B

I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

Student C

I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even.



Student D

I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number.

Student E

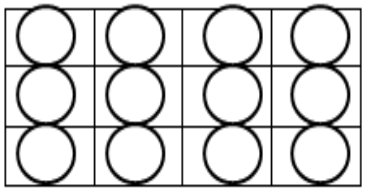
I know that 4 plus 4 equals 8. So 8 is an even number.

The focus of this standard is placed on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation and should not be used as the definition of an even number. (Van de Walle & Lovin, 2006, p. 292)

2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends

Second graders use rectangular arrays to work with repeated addition, a building block for multiplication in third grade. A rectangular array is any arrangement of things in rows and columns, such as a rectangle of square tiles. Students explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

Example: **What is the total number of circles below?**



Student A	**Student B**
I see 3 counters in each column and there are 4 columns. So I added 3 + 3 + 3 + 3. That equals 12.	I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.
3 + 3 + 3 + 3 = 12	4 + 4 + 4 = 12

2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

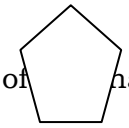
In Second Grade, students pose a question, determine up to 4 categories of possible responses, collect data, represent data on a picture graph or bar graph, and interpret the results. This is an extension from first grade when students organized, represented, and interpreted data with up to three categories. They are able to use the graph selected to note particular aspects of the data collected, including the total number of responses, which category had the most/least responses, and interesting differences/similarities between the four categories. They then solve simple one-step problems using the information from the graph.

2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **[5] Sizes are compared directly or visually, not compared by measuring**

Second Grade students identify (recognize and name) shapes and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

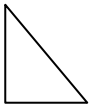
Example:

Teacher: Draw a closed shape that has five sides. What is the name of this shape?



Student: I drew a shape with 5 sides. It is called a pentagon.

Example:



Teacher: I have 3 sides and 3 angles. What am I?

Student: A triangle. See, 3 sides, 3 angles.

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides**. With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.).

2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them

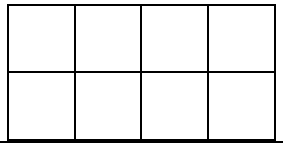
Second graders partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This work connects to the standard 2.OA.4.

Where students are arranging objects in an array of rows and columns. This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication

Example:

T**eacher**: Partition the rectangle into 2 rows and 4 columns. How many small squares did you make?

Student: There are 8 squares in this rectangle. See- 2, 4, 6, 8. I folded the paper to make sure that they were all the same size.



	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	**Chapter Opener** Assessing Prior Knowledge *The Pre Test serves as a diagnostic test of readiness of the upcoming chapter*	**Teacher Materials** Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge **Student Materials** Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge	Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper Quick Check and the Pre Test can be done in the same block *(See Anecdotal Checklist; Transition Guide)* Recall Prior Knowledge – Quick Check – Pre Test
DIRECT ENGAGEMENT	**Direct Involvement/Engagement** Teach/Learn *Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions*	**Teacher Edition** 5-minute warm up Teach; Anchor Task **Technology** Digi **Other** Fluency Practice	• The Warm Up activates prior knowledge for each new lesson • Student Books are CLOSED; Big Book is used in Gr. K • Teacher led; Whole group • Students use concrete manipulatives to explore concepts • A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning • Teacher facilitates; Students find the solution
GUIDED LEARNING	**Guided Learning and Practice** Guided Learning	**Teacher Edition** Learn **Technology** Digi **Student Book** Guided Learning Pages Hands-on Activity	Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking **Small Group w/Teacher circulating among groups** Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	**Independent Practice** *A formal formative assessment*	**Teacher Edition** Let's Practice **Student Book** Let's Practice **Differentiation Options** All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	**Let's Practice** determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives **CAN** be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
ADDITIONAL PRACTICE	**Extending the Lesson**	Math Journal Problem of the Lesson Interactivities Games	
	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	**End of Chapter Wrap Up and Post Test**	**Teacher Edition** Chapter Review/Test Put on Your Thinking Cap **Student Workbook** Put on Your Thinking Cap **Assessment Book** Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed • Individually (e.g. for homework) then reviewed in class • As a 'mock test' done in class and doesn't count • As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.

Misconceptions:

- Students may create patterns in which numbers that end with 0,2,4,6, or 8 are even and odd numbers end in 1,3,5,7, or 9 but they do not define or provide a conceptual understanding of what an even or odd number is.

- Students may not think of a configuration with 1 row or 1 colum as an array.

- Some students may forget to label and title the graphs or have difficulty reading a graph. This leads to confusion in analyzing the data in their graphs. Teachers can begin by helping students count the number in each category.

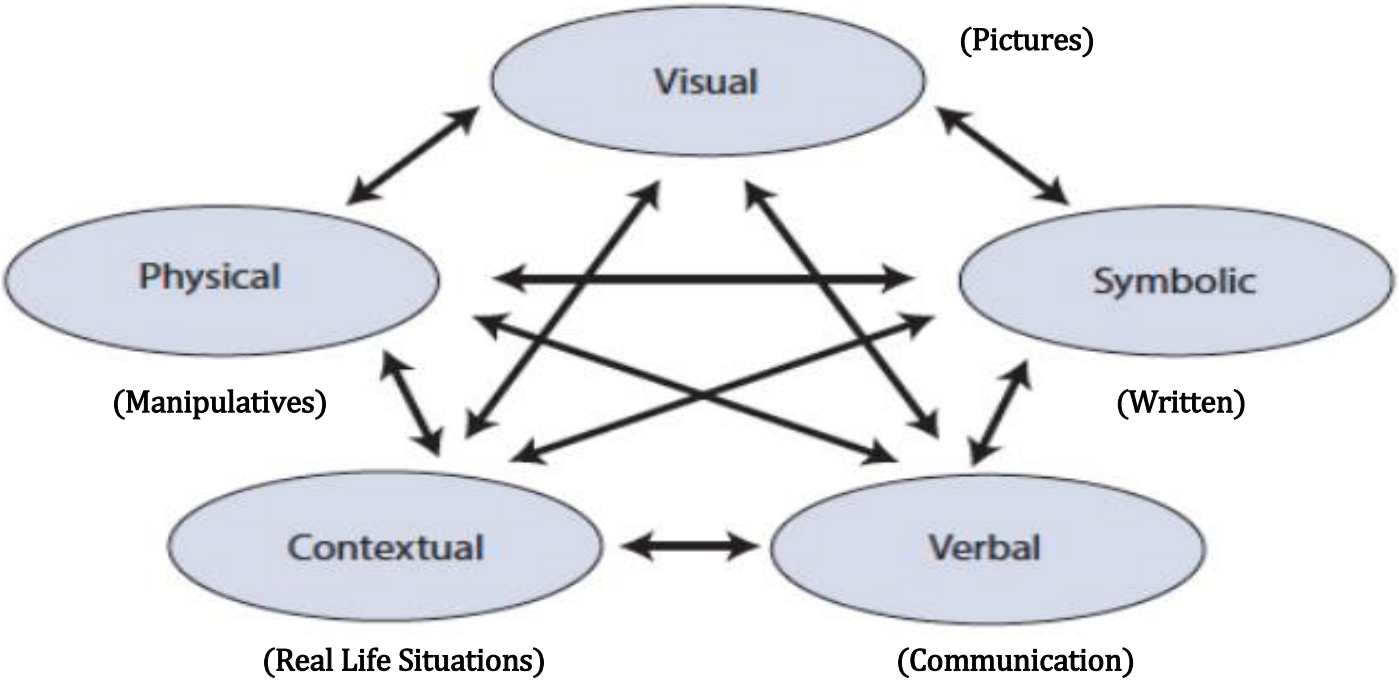
- Students may forget that a symbol can be used to represent 1 or more depending on the key.

- Students may think that a shape changes based on it's orientation. It is essential to give them the opportunity to touch and feel the shapes to deiscover that the shape does not change.

- Assure that students have hands on shapes so that they are able to trace shapes and eventually sketch them when needed.

- Students may confuse a row and a column. Continue to enforce vocabulary when discussing arrays.

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

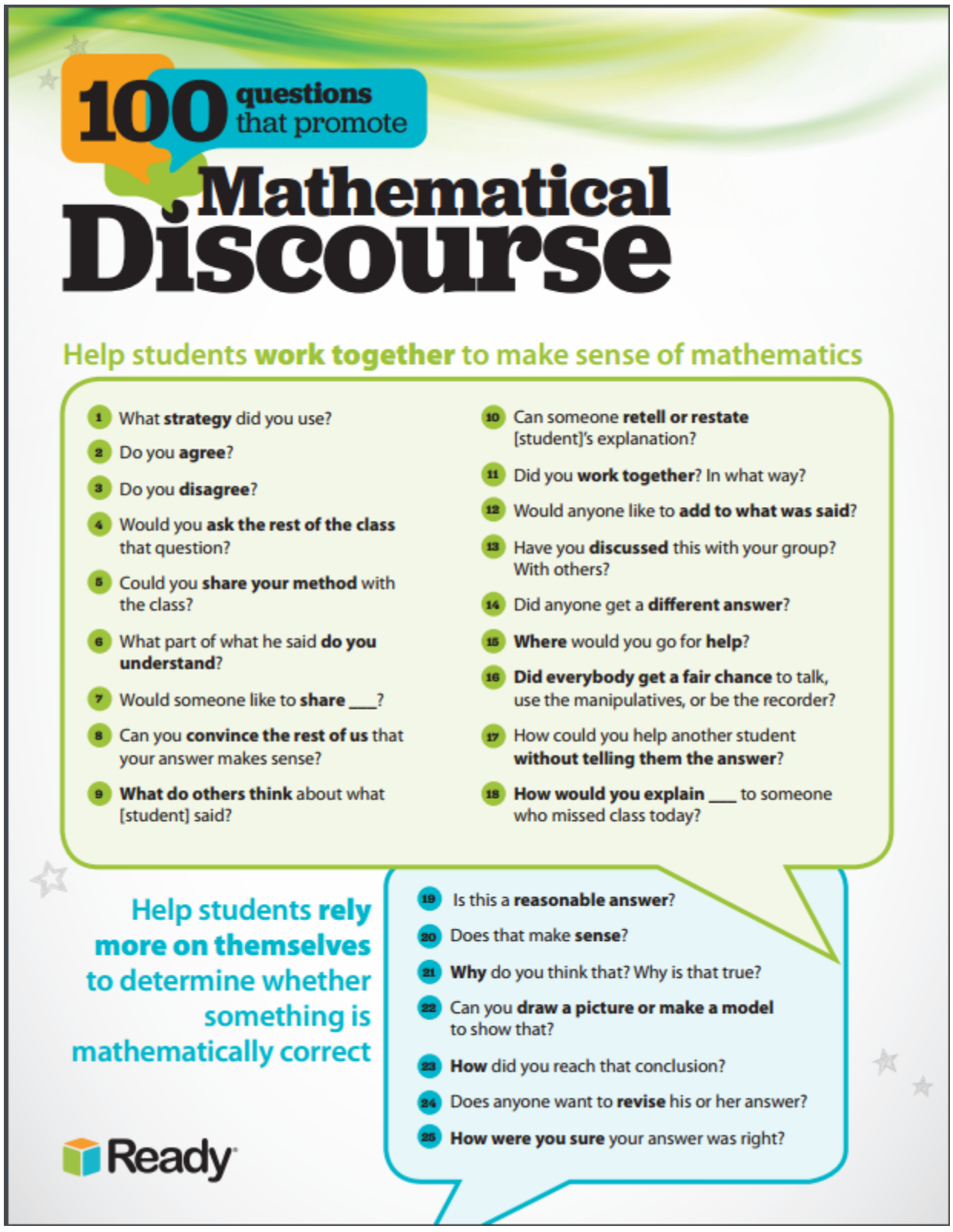
Discourse involves asking strategic questions that elicit from students both how a problem was solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



Help students learn to **reason mathematically**

26. **How did you begin** to think about this problem?
27. What is **another way** you could solve this problem?
28. How could you **prove** _____?
29. Can you **explain how your answer is different from or the same as** [student]'s answer?
30. Let's **break the problem into parts**. What would the parts be?
31. Can you **explain this part more specifically**?
32. Does that **always work**?
33. Can you think of a case where that **wouldn't work**?
34. How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

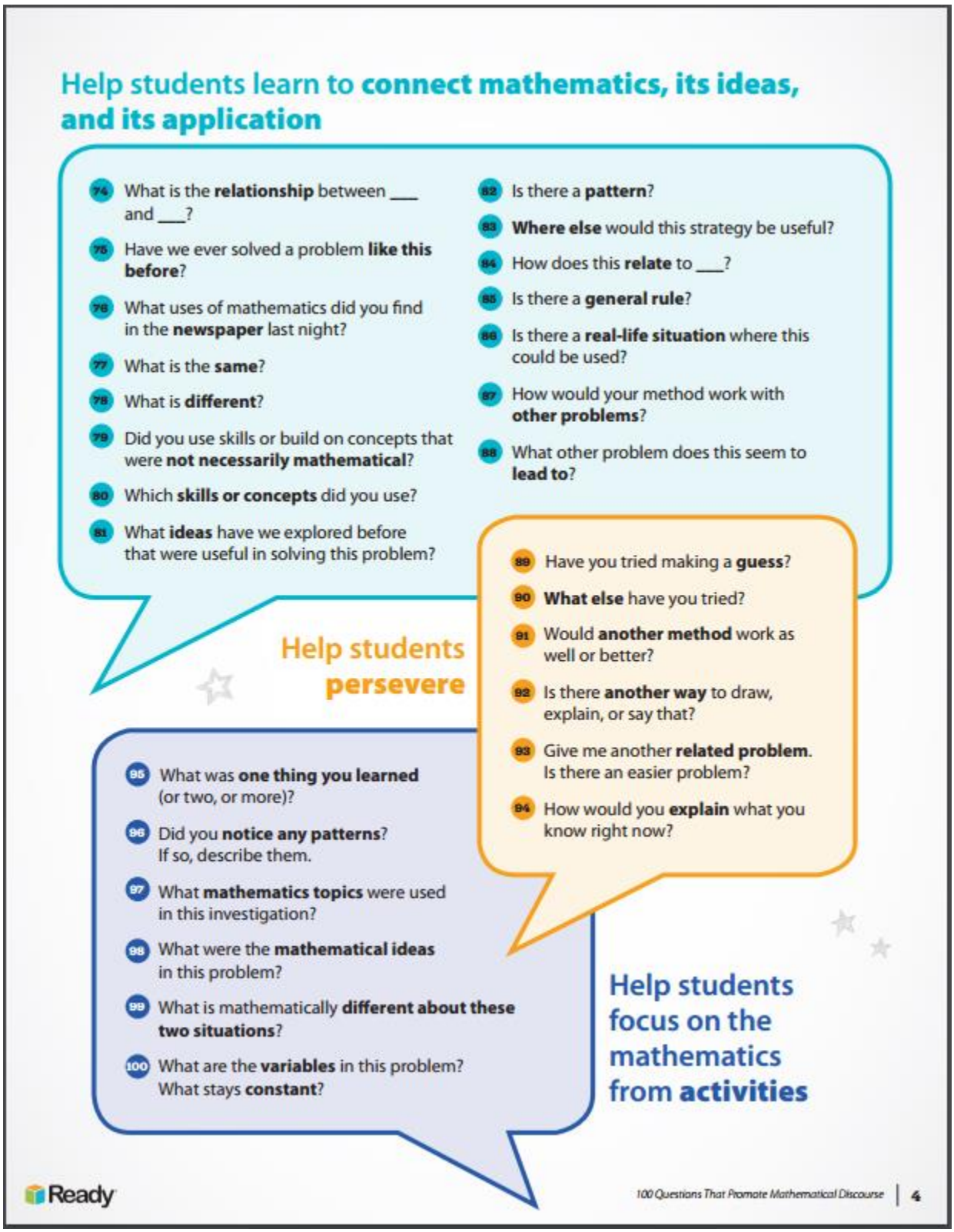
35. What do you need to do **next**?
36. What have you **accomplished**?
37. What are your **strengths and weaknesses**?
38. Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39. What is this problem about? What can you **tell me about it**?
40. Do you need to **define or set limits** for the problem?
41. How would you **interpret** that?
42. Could you **reword that in simpler terms**?
43. Is there something that can be **eliminated** or that is **missing**?
44. Could you **explain** what the problem is asking?
45. What **assumptions** do you have to make?
46. What do you **know** about this part?
47. Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48. **What would happen if ___?**
49. Do you see a **pattern**?
50. What are some **possibilities** here?
51. Where could you find the **information** you need?
52. How would you **check your steps** or your answer?
53. What **did not work**?
54. How is your solution method the **same as or different from** [student]'s method?
55. Other than retracing your steps, **how can you determine** if your answers are appropriate?
56. How did you **organize** the information? Do you have a **record**?
57. How could you solve this using **tables, lists, pictures, diagrams**, etc.?
58. What have you tried? What **steps** did you take?
59. How would it look if you used this **model** or these **materials**?
60. How would you draw a **diagram or make a sketch** to solve the problem?
61. Is there **another possible answer**? If so, explain.
62. Is there **another way to solve** the problem?
63. Is there **another model** you could use to solve the problem?
64. Is there anything you've **overlooked**?
65. **How did you think** about the problem?
66. What was your **estimate or prediction**?
67. How **confident** are you in your answer?
68. **What else** would you like to know?
69. What do you think comes **next**?
70. Is the solution **reasonable**, considering the context?
71. Did you have a **system**? Explain it.
72. Did you have a **strategy**? Explain it.
73. Did you have a **design**? Explain it.


Help students learn to connect mathematics, its ideas, and its application
74 What is the relationship between ___ and ___?
75 Have we ever solved a problem like this before?
76 What uses of mathematics did you find in the newspaper last night?
77 What is the same?
78 What is different?
79 Did you use skills or build on concepts that were not necessarily mathematical?
80 Which skills or concepts did you use?
81 What ideas have we explored before that were useful in solving this problem?
82 Is there a pattern?
83 Where else would this strategy be useful?
84 How does this relate to ___?
85 Is there a general rule?
86 Is there a real-life situation where this could be used?
87 How would your method work with other problems?
88 What other problem does this seem to lead to?
Help students persevere
89 Have you tried making a guess?
90 What else have you tried?
91 Would another method work as well or better?
92 Is there another way to draw, explain, or say that?
93 Give me another related problem. Is there an easier problem?
94 How would you explain what you know right now?
95 What was one thing you learned (or two, or more)?
96 Did you notice any patterns? If so, describe them.
97 What mathematics topics were used in this investigation?
98 What were the mathematical ideas in this problem?
99 What is mathematically different about these two situations?
100 What are the variables in this problem? What stays constant?
Help students focus on the mathematics from activities
Ready
100 Questions That Promote Mathematical Discourse | 4

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.
1.OA.6 Add and Subtract within 10.
2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.
Use strategies such as:

- counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$);
- decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

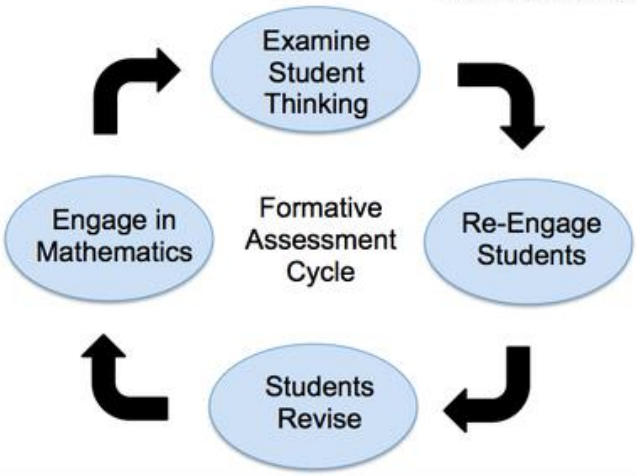
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lesson

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)



Unit 4 Assessment / Authentic Assessment Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 6			
Mid- Module Assessment	2.OA.4	1 Block	Individual
End of Module Assessment	2.OA.3 2.OA.4 2.G.2	1 Block	Individual
Chapter 17			
Optional Chapter 17 Test	2.MD.8	1 block	Individual
Authentic Assessment: Show Money Amounts	2.MD.8	½ block	Individual
Chapter 19			
Optional Chapter 19 Test	2.G.2-3	1 block	Individual
Grade 2 Interim Assessment 4	2.NBT.6-9 2.G.2-3 2.MD.7-8	1 Block	Individual

	PLD	**Genesis Conversion**
Rubric Scoring	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	
1	**Make sense of problems and persevere in solving them** Mathematically proficient students in Second Grade examine problems and tasks, can make sense of the meaning of the task and find an entry point or a way to start the task. Second Grade students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In Second Grade, students' work continues to use concrete manipulatives and pictorial representations as well as mental mathematics. Second Grade students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"
2	**Reason abstractly and quantitatively** Mathematically proficient students in Second Grade make sense of quantities and relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Second Grade, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria and they are joined by 17 more children. How many students are in the cafeteria? " Second Grade students translate that situation into an equation, such as: 25 + 17 = __ and then solve the problem. Students also contextualize situations during the problem solving process. For example, while solving the task above, students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The processes of reasoning also other areas of mathematics such as determining the length of quantities when measuring with standard units.

3	**Construct viable arguments and critique the reasoning of others**
	Mathematically proficient students in Second Grade accurately use definitions and previously established solutions to construct viable arguments about mathematics. During discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. For example, while solving 74 - 18, students may use a variety of strategies, and after working on the task, can discuss and critique each other's reasoning and strategies, citing similarities and differences between strategies.
4	**Model with mathematics**
	Mathematically proficient students in Second Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Second Grade students use concrete manipulatives and pictorial representations to provide further explanation of the equation. Likewise, Second Grade students are able to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation 43 + 17 = ___ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"
5	**Use appropriate tools strategically**
	Mathematically proficient students in Second Grade have access to and use tools appropriately. These tools may include snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students also have experiences with educational technologies, such as calculators and virtual manipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction, students have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.
6	**Attend to precision**
	Mathematically proficient students in Second Grade are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Second Grade communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
7	**Look for and make use of structure**
	Mathematically proficient students in Second Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, students notice number patterns within the tens place as they connect skip count by 10s off the decade to the corresponding numbers on a 100s chart. While working in the Numbers in Base Ten domain, students work with the idea that 10 ones equal a ten, and 10 tens equals 1 hundred. In addition, Second Grade students also make use of structure when they work with subtraction as missing addend problems, such as 50- 33 = __ can be written as 33+ __ = 50 and can be thought of as," How much more do I need to add to 33 to get to 50?"

	Look for and express regularity in repeated reasoning
8	Mathematically proficient students in Second Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomposing numbers (33+ 25 = 30 + 20 + 3 +5), students may begin to generalize and frequently apply that strategy independently on future tasks. Further, students begin to look for strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving all tasks, Second Grade students accurately check for the reasonableness of their solutions during and after completing the task.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

1st & 2nd Grade Ideal Math Block

Essential Components

FLUENCY: Partner/Small Group — 5 min.

CONCRETE, PICTORIAL, and **ABSTRACT** approaches to support **ARITHMETIC FLUENCY** and **FLUENT USE OF STRATEGIES.**

LAUNCH: Whole Group — 15-20 min.

Anchor Task: Math In Focus Learn

EXPLORATION: Partner / Small Group

Math In Focus Hands-On, Guided Practice, Let's Explore

INDEPENDENT PRACTICE: Individual

Math In Focus Let's Practice, Workbook, Reteach, Extra Practice, Enrichment

MATH WORKSTATIONS: Pairs / Small Group/ Individual — 15-20 min.

DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

Small Group Instruction	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal Lab

SUMMARY: Whole Group

Lesson Closure: Student Reflection; Real Life Connections to Concept

EXIT TICKET (DOL): Individual — 5 min.

Students complete independently; Used to guide instructional decisions; Used to set instructional goals for students;

Note:

- Place emphasis on the flow of the lesson in order to ensure the development of students' conceptual understanding.
- Outline each essential component within lesson plans.
- Math Workstations may be conducted in the beginning of the block in order to utilize additional support staff.
- Recommended: 5-10 technology devices for use within **TECHNOLOGY** and **FLUENCY** workstations.

Second Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% **Distinguished command**	**PLD Level 4: 89%** **Strong Command**	**PLD Level 3: 79%** **Moderate Command**	**PLD Level 2: 69%** **Partial Command**	**PLD Level 1: 59%** **Little Command**

Student work shows **distinguished levels of understanding** of the mathematics.
 	
Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:
• Tools:
  ○ Manipulatives
  ○ Five Frame
  ○ Ten Frame
  ○ Number Line
  ○ Part-Part-Whole Model
• Strategies:
  ○ Drawings
  ○ Counting All
  ○ Count On/Back
  ○ Skip Counting
  ○ Making Ten
  ○ Decomposing Number
• Precise use of math vocabulary
Response includes an **efficient and logical progression** of mathematical reasoning and understanding.
	Student work shows **strong levels of understanding** of the mathematics.
	
	Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:
	• Tools:
	  ○ Manipulatives
	  ○ Five Frame
	  ○ Ten Frame
	  ○ Number Line
	  ○ Part-Part-Whole Model
	• Strategies:
	  ○ Drawings
	  ○ Counting All
	  ○ Count On/Back
	  ○ Skip Counting
	  ○ Making Ten
	  ○ Decomposing Number
	• Precise use of math vocabulary
	
	Response includes a **logical progression** of mathematical reasoning and understanding.
		Student work shows **moderate levels of understanding** of the mathematics.
		
		Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:
		• Tools:
		  ○ Manipulatives
		  ○ Five Frame
		  ○ Ten Frame
		  ○ Number Line
		  ○ Part-Part-Whole Model
		• Strategies:
		  ○ Drawings
		  ○ Counting All
		  ○ Count On/Back
		  ○ Skip Counting
		  ○ Making Ten
		  ○ Decomposing Number
		• Precise use of math vocabulary
		
		Response includes a **logical but incomplete progression** of mathematical reasoning and understanding.
		Contains **minor errors**.
			Student work shows **partial understanding** of the mathematics.
			
			Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the:
			• Tools:
			  ○ Manipulatives
			  ○ Five Frame
			  ○ Ten Frame
			  ○ Number Line
			  ○ Part-Part-Whole Model
			• Strategies:
			  ○ Drawings
			  ○ Counting All
			  ○ Count On/Back
			  ○ Skip Counting
			  ○ Making Ten
			  ○ Decomposing Number
			• Precise use of math vocabulary
			
			Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.
				Student work shows **little understanding** of the mathematics.
				
				Student **attempts** to **constructs** and **communicates** a response using the:
				• Tools:
				  ○ Manipulatives
				  ○ Five Frame
				  ○ Ten Frame
				  ○ Number Line
				  ○ Part-Part-Whole Model
				• Strategies:
				  ○ Drawings
				  ○ Counting All
				  ○ Count On/Back
				  ○ Skip Counting
				  ○ Making Ten
				  ○ Decomposing Number
				• Precise use of math vocabulary
				
				Response includes **limited evidence of the progression** of mathematical reasoning and understanding.

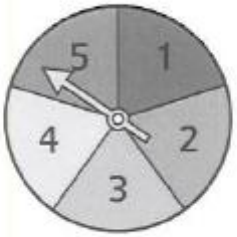
5 points	**4 points**	**3 points**	**2 points**	**1 point**

2nd Grade Authentic Assessment: Playing Games

A game company wants to create a new game. They are trying to figure out some of the rules of the game.

Part A

They want to use a spinner to distribute red and blue coins equally. If the spinner lands on an even number, the player gets a red coin. If the spinner lands on an odd number, the player gets a blue coin. If the game uses the spinner shown, how many numbers will result in a red coin? How many will result in a blue coin? Explain.



Part B

Will the spinner in Part A give the same number of red and blue coins? Explain.

Part C

Suppose gray coins are worth 3 points and black coins are worth 2 points. Write and solve an equation that shows how many points the gray coins are worth. Write and solve another equation that shows how many points the black coins are worth. Then write how many total points the player has.

Authentic Assessment: Playing Games Rubric

Part A	
Score	**Description**
2	Student response includes the following 2 elements • **Number of Red/Blue Coins:** 1 point • **Explanation:** 1 point Sample Student Response: 2 numbers will result in an even number and a red coin: 2, 4 3 numbers will result in an odd number and a blue coin: 1, 3, 5
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

Part B	
Score	**Description**
2	Student response includes the following 2 elements • **Yes/No:** 1 point • **Explanation:** 1 point Sample Student Response: No, since there are 3 options for blue coins and 2 options for red coins, there will be more options for blue coins to be given than red coins.
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

Part C	
Score	**Description**
3	Student response includes the following 3 elements • **Gray Coins:** 1 point • **Black Coins:** 1 point • **Total:** 1 point Sample Student Response: Gray: 3 + 3 = 6 points Black: 2 + 2 + 2 + 2 = 8 points Total: 6 + 8 = 14 points
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000
http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx
Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center: http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com